Vision Research 187 (2021) 110–119

Contents lists available at ScienceDirect

Vision Research

journal homepage: www.elsevier.com/locate/visres

The effect of response biases on resolution thresholds of Sloan letters in central and paracentral vision

Hatem Barhoom [*], Mahesh R. Joshi, Gunnar Schmidtmann

Eye and Vision Research Group, School of Health Professions, University of Plymouth, Plymouth, UK

ARTICLE INFO

Keywords:
Visual acuity
Letter acuity
Resolution threshold
Response bias
Luce's choice model
Sloan letters

ABSTRACT

Sloan letters are one of the most commonly used optotypes in clinical practice. Sloan letters have different relative legibility which could be due to three factors: perceivability, response bias, and similarity. Similarities between Sloan letters are known to be the major source of errors in threshold determination. However, little is known about the effect of response biases on the resolution thresholds. The aim of the present study was to investigate the effect of response bias and similarity on resolution thresholds of Sloan letters in central and paracentral vision.

Eight subjects with normal ocular health participated in this study. Using the method of constant stimuli, we measured resolution thresholds for the Sloan letters set at 0° (central) and ± 3° eccentricity along the vertical meridian of the visual field. We calculated thresholds from data pooled across the 10 Sloan letters (pooled threshold). For further analysis we also calculated thresholds for each of the 10 Sloan letters (individual threshold). Response biases and letter similarities were determined using Luce's choice model.

Results showed statistically significant differences between the mean individual thresholds of Sloan letters at the central and the upper visual field, but not at the lower visual field. For equally-sized letters at pooled threshold, unlike letter similarity, response biases showed statistically significant correlations to the differences in individual thresholds at the central, upper and lower visual field locations. For equally legible letters at individual thresholds, response biases and similarities showed no significant correlations to the differences in individual thresholds at the central, the upper and the lower visual field locations.

These results suggest that, for equally-sized letters at pooled threshold, the response biases may lead to an underestimation of the pooled threshold, *i.e.* an overestimation of visual acuity measurements when using Sloan letters.

1. Introduction

Visual acuity is the ability of the visual system to discern the smallest details of an object, typically measured as the minimum angle of resolution (*i.e.* detection/resolution threshold) and is of high clinical importance. Clinically, different stimuli or optotypes, such as the Tumbling E, Landolt C and alphanumeric characters have been employed to measure visual acuity (Kniestedt & Stamper, 2003). Although the Landolt C is internationally regarded as the reference optotype (Sloan, 1959; Treacy et al., 2015), letters are used in many visual acuity charts, because they are intuitive and easy to use in clinical settings. Furthermore, employing a variety of letters (*e.g.* Sloan letters) reduces the guessing rate associated with forced-choice tests (Pelli & Robson, 1991).

Letter identification accuracy is influenced by three factors: (i) perceivability, (ii) response bias and (iii) similarity (Mueller & Weidemann, 2012). Perceivability is a measure of how legible the letter is depending solely on the characteristics of the letters, such as the letter size, contrast or shape. The response bias is defined as the tendency of favouring one response over the other alternatives (Macmillan & Creelman, 1990). Similarity is defined as the confusion in letter perception which arises among certain letters. In other words, letter recognition, *i.e.* the letter detection/resolution threshold, could be affected by changing the amount or the type of the sensory input, *e.g.* size and contrast (perceivability), the bias towards certain letters in case of uncertainty (response biases), and the confusion between similar letters such as C and O (similarity). Note that from these definitions it is well understood that response biases, unlike perceivability and letter similarities, are

---

[*] Corresponding author.
E-mail address: hatem.barhoom@plymouth.ac.uk (H. Barhoom).

https://doi.org/10.1016/j.visres.2021.06.002
Received 4 September 2020; Received in revised form 30 May 2021; Accepted 8 June 2021

independent of the sensory inputs of the stimulus.

It has been demonstrated that letters have different legibilities at the fovea (Grimm, Rassow, Wesemann, Saur, & Hilz, 1994; Alexander, Xie, & Derlacki, 1997; Reich & Bedell, 2000; Shah, Dakin, & Anderson, 2012; Hamm, Yeoman, Anstice, & Dakin, 2018; Ludvigh, 1941; Strasburger, Rentschler, & Juttner, 2011; Hairol et al., 2015) and peripheral visual field locations (Ludvigh, 1941; Strasburger et al., 2011; Hairol et al., 2015; Anderson & Thibos, 2004; Shah, Dakin, Redmond, & Anderson, 2011; Shah et al., 2012). The focus of most previous research was to investigate the relative legibility of letters in order to determine the most equally legible letters to optimise the design of letter optotypes used in visual acuity charts. The acceptable differences in legibility between letters is determined by the International Standard ISO 8597. According to this standard, the visual acuity measured by a full set of letters should not deviate by more than 0.05 log units from the visual acuity measured with the Landolt C chart. However, Grimm et al. (1994) recommended that the resolution thresholds of each individual letter should be within 0.05 log units from the mean resolution threshold of the letters set (Grimm et al., 1994). This is of particular importance in the case of letter-by-letter visual acuity measurements. It has been shown that further improvement of the precision of the visual acuity measurements using letters can be obtained by weighting the responses to the letters according to their individual legibility and similarity (Grimm et al., 1994; Mcmonnies & Ho, 1996; McMonnies & Ho, 2000).

Unlike other common letter stimuli, Sloan letters have been adopted in the design of various letter charts (*e.g.* Early Treatment Diabetic Retinopathy Study; ETDRS chart), because their average legibility, determined by the letter identification accuracy, is similar to the Landolt C (Sloan, 1959; Treacy et al., 2015). It has been shown that Sloan letters have different relative legibility at the fovea where the letter similarities are the major source of errors in threshold determination (Mcmonnies & Ho, 1996; Reich & Bedell, 2000; Hamm et al., 2018).

However, little is known about the effect of response biases on the resolution thresholds (or legibility) of individual Sloan letters. In this study, we aim to investigate the relationship between the response biases, similarity and resolution thresholds of individual Sloan letters in central and paracentral locations since the pattern of differences in letter thresholds has been found to be different at the central and paracentral locations (Ludvigh, 1941; Strasburger et al., 2011; Hairol et al., 2015).

2. Methods

2.1. Participants

Eight naïve subjects (six females, mean age 22.60 ± 3.70 (SD), age range:19–28 years) with normal ocular health participated in this study. The mean best corrected visual acuity and the mean refractive error (spherical equivalent) were −0.04 ± 0.05 logMAR and −2.30 ± 2.63 *DS* respectively. All tests were conducted monocularly (left or right eye, chosen at random), where the fellow eye was occluded using an opaque eye patch. Written informed consent was obtained from all observers, and the study was approved by the University of Plymouth Ethics committee. All experiments were conducted in accordance with the Declaration of Helsinki.

2.2. Apparatus

Stimuli were generated using MATLAB R2016b (MathWorks, Natick, Massachusetts, USA). Routines from the Psychtoolbox-3 were used to present the stimuli (Brainard, 1997; Pelli, 1997; Kleiner, Brainard, Pelli, Ingling, Murray, & Broussard, 2007). The stimuli were presented on a gamma-corrected DELL, P2317H LCD monitor (1920 × 1080) with a frame rate of 60 Hz. Monitor linearization was achieved by adjusting colour look-up tables, resulting in 150 approximately equally 2 spaced grey levels. Observers viewed the targets at a viewing distance of 350 cm, while sitting on chair without using a chin or forehead rest. The examiner monitored viewing distance by regular checks. At this viewing distance one pixel subtended 0.258 min of arc (′) of visual angle. Experiments were carried out under a room illumination of 160 lx. The observer responded by calling out the responses which were entered by the experimenter via a standard computer keyboard. This method minimised errors caused by mistyping and improved fixation compliance.

2.3. Stimuli

High contrast Sloan letters were used in the experiments (black letters of 2.2 cd/m^2 on a white background of 215 cd/m^2, resulting in 99% Weber contrast). The variables were letter size, expressed in minutes of arc, and the location of presentation. The letters were presented centrally and at paracentral locations along the vertical meridian at an eccentricity of 3° in the upper and lower visual field (Fig. 1a). Ten standard Sloan letters (C, D, H, K, N, O, R, S, V, Z) were used. Sloan letters are designed so that their height is equal to their width and five times the stroke width (Fig. 1b). We conducted multiple pilot experiments to establish appropriate stimulus levels (letter sizes) to cover the whole range of responses (from chance (10%) to certain decision (100%)). Six different letter sizes (spaced logarithmically) were tested; 0.3′, 0.44′, 0.64′, 0.94′, 1.37′, and 2′ for central presentations and 0.5′, 0.79′, 1.26′, 1.99′, 3.15′ and 5′ for paracentral presentations.

2.4. Procedure

The method of constant stimuli was used in all experiments in this study. The Sloan letters were presented randomly across three locations. Each subject completed 1800 trials for the full experiment (six letter sizes × three locations × 10 Sloan letters × 10 presentations per letter). All conditions were interleaved. The presentation time was 250 ms and presentations were accompanied by an auditory signal. The task for the observer was to recognise the presented letter and to report it verbally. During the experiment, the subjects were asked to fixate on a fixation cross (dimensions: length/width 1.55′, stroke width 0.036′) presented at the centre of the screen. The fixation cross disappeared for the duration of the central presentations and reappeared for the paracentral presentations. Subjects were encouraged to guess when uncertain about the letter. Only choices of the 10 Sloan letters were accepted. To familiarise the participants with the Sloan letters, the experimenter demonstrated the Sloan letters in the beginning of the session. The observers showed excellent compliance to answer from the Sloan letter set (on average not more than 30 mistakes per subject). In the rare case where observers responded with a non-Sloan letters, the experimenter prompted for a second response. If the observer failed the second attempt, a reminder of the Sloan letter set was provided (this occurred very rarely, on average

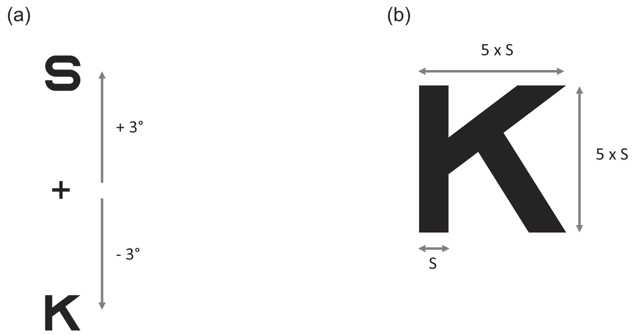


Fig. 1. (a) Sloan letters presented centrally, or along the vertical meridian at an eccentricity of 3° in the upper and lower visual field. The Sloan letters S and K are shown for illustration purposes (not to scale). (b) shows the dimensions of the Sloan letters, exemplified by the letter K. The stroke width $S = 1/5$ of the letter's height. The height and the width of the letter are equal.

not more than once per subject).

2.5. Analysis

Routines from the Palamedes Toolbox (Prins & Kingdom, 2018) were employed to fit individual psychometric functions. Gumbel (Log-Weibull) functions (Eq. (1)) were fit to determine thresholds.

$$P_{correct} = (\gamma + (1 - \gamma - \lambda))\left(1 - \exp\left(-10^{(\sigma(x-\alpha))}\right)\right) \quad (1)$$

where γ is the guessing rate $(10\ \mathit{letters}, \gamma = 0.1)$, λ is the lapse rate $(\lambda = 0.02, \text{naïve subjects})$, x is the letter size (log visual angle), α is the threshold and σ is the slope of the function. The threshold α was defined as x yielding 65.6% correct responses, according to the following equation.

$$P_{correct} = 0.1 + (1 - 0.1 - 0.02)\left(1 - \exp\left(-10^{(\sigma(\alpha-\alpha))}\right)\right) \approx 0.656 \quad (2)$$

Thresholds were calculated from data pooled across the 10 Sloan letters. We refer to these as pooled thresholds. For further analysis we also calculated thresholds for each of the 10 Sloan letters. These are referred to as individual thresholds.

3. Results

The mean pooled thresholds across subjects (±SD) were 0.04 ±0.05, 0.40 ±0.11 and 0.42 ±0.12 log visual angle at the central, upper visual field (3°) and lower visual field (3°) locations respectively. Paired sample *t*-test revealed no statistically significant differences between the mean pooled thresholds at upper and lower visual field locations $(t\ (7) = -0.77,\ p = .47)$. However, the mean pooled thresholds at the upper visual field $(t\ (7) = 6.12, p < .001)$ and lower visual field $(t\ (7) = 6.90, p < .001)$ were significantly higher compared to the central location. The mean individual thresholds across subjects at each location are shown in Table 1. One-way ANOVA tests showed statistically significant differences between the mean individual thresholds at the central location $(F\ (9, 70) = 4.93, p < .001)$ (with highest and lowest thresholds for O = 0.10 ±0.06 and V = −0.10 ±0.05 respectively) and upper visual field $(F\ (9, 70) = 2.0, p < .05)$ (with highest and lowest thresholds for S = 0.46 ± 0.18 and H = 0.25± 0.14 respectively). However, there was no statistically significant difference between the mean individual thresholds at the lower visual field $(F\ (9, 70) = 1.7, p = .057)$ (with highest and lowest thresholds for O = 0.52 ± 0.20 and V = 0.29 ± 0.13 respectively).

In order to investigate the pattern of the individual threshold differences, we calculated the relative thresholds for the letters as the difference between the pooled thresholds and the individual thresholds (Fig. 2). The analysis revealed statistically significant correlations of relative thresholds between the lower visual field and the central location $(r = 0.83, n = 10, p < .05)$ and also between the lower visual field and upper visual field $(r = 0.72, n = 10, p < .05)$. This suggests that the pattern of differences between individual thresholds are consistent at the central location, and the upper and lower visual field (Fig. 3).

For the following analyses, confusion matrices (presented vs. responses of letters) were created for individual observers at each location. Note that the data in the current study were not collected at the pooled threshold or the individual threshold sizes. We therefore extrapolated the expected confusion matrices from the available data (see Appendix for details).

Table 1
The table shows the mean of individual thresholds (Mean ± SD in log visual angle) at central and paracentral locations.

	Central	Upper field (3°)	Lower field (3°)
C	0.06 ± 0.11	0.377 ± 0.08	0.45 ± 0.14
D	0.04 ± 0.09	0.40 ± 0.13	0.42 ± 0.12
H	0.02 ± 0.11	0.25 ± 0.14	0.34 ± 0.10
K	0.06 ± 0.07	0.40 ± 0.09	0.43 ± 0.07
N	−0.07 ± 0.08	0.32 ± 0.14	0.36 ± 0.16
O	0.10 ± 0.06	0.44 ± 0.17	0.52 ± 0.20
R	−0.02 ± 0.06	0.33 ± 0.17	0.38 ± 0.14
S	0.06 ± 0.10	0.46 ± 0.18	0.39 ± 0.17
V	−0.10 ± 0.05	0.27 ± 0.07	0.29 ± 0.13
Z	−0.01 ± 0.02	0.35 ± 0.11	0.32 ± 0.10

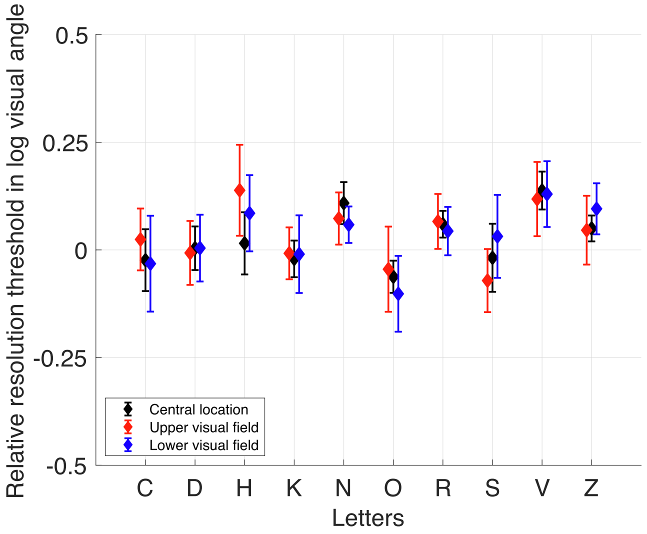


Fig. 2. shows the mean relative thresholds of letters at central, upper (3°) and lower visual field (3°). The error bars here and throughout represent 95% confidence intervals.

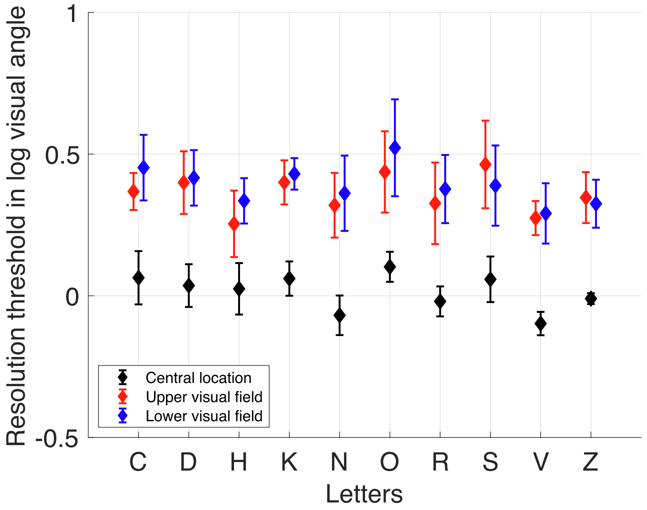
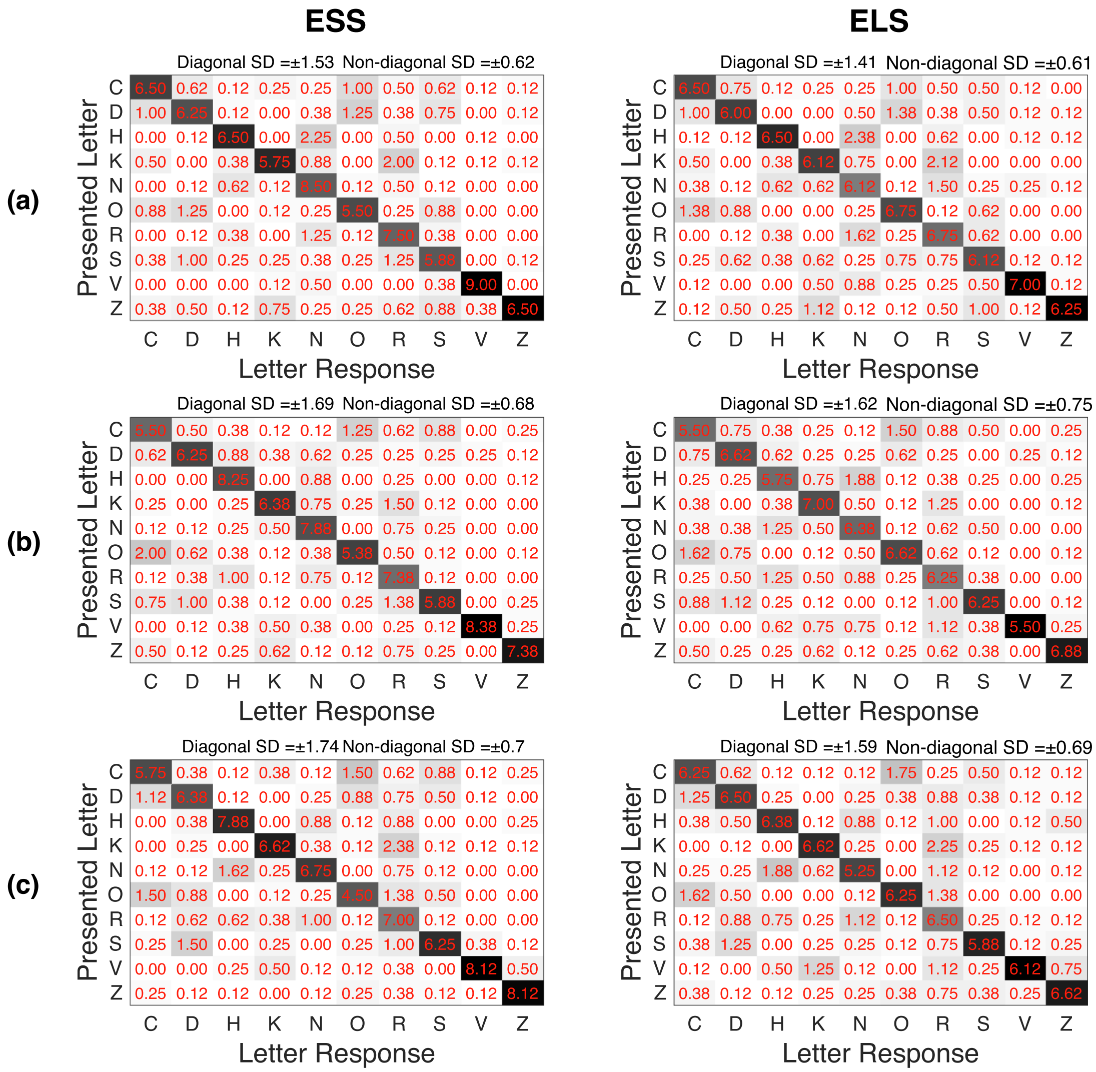


Fig. 3. shows the mean individual thresholds at central, upper and lower visual field locations.

The confusion matrices were calculated for two conditions. In the first condition, the expected confusion matrices at each subject's pooled threshold size were calculated. As a consequence, all presented letters in this condition had equal stroke sizes (referred to as ESS) which was equal to the size of the pooled threshold. In the second condition, the expected confusion matrices at each letter's individual threshold sizes were calculated. In this case all letters had equal legibility sizes (referred to as ELS), regardless of the stroke size of the letter (equal performances; $P_{correct} \approx 0.656$). Fig. 4 shows the mean (±SD) of the expected confusion matrices for the two conditions (ESS and ELS) at central, upper and lower visual field.

3.1. Model

The difference in individual thresholds between letters can be caused



(a)

ESS

Diagonal SD =±1.53 Non-diagonal SD =±0.62

	C	D	H	K	N	O	R	S	V	Z
C	6.50	0.62	0.12	0.25	0.25	1.00	0.50	0.62	0.12	0.12
D	1.00	6.25	0.12	0.00	0.38	1.25	0.38	0.75	0.00	0.12
H	0.00	0.12	6.50	0.00	2.25	0.00	0.50	0.00	0.12	0.00
K	0.50	0.00	0.38	5.75	0.88	0.00	2.00	0.12	0.12	0.12
N	0.00	0.12	0.62	0.12	8.50	0.12	0.50	0.12	0.00	0.00
O	0.88	1.25	0.00	0.12	0.25	5.50	0.25	0.88	0.00	0.00
R	0.00	0.12	0.38	0.00	1.25	0.12	7.50	0.38	0.00	0.00
S	0.38	1.00	0.25	0.25	0.38	0.25	1.25	5.88	0.00	0.12
V	0.00	0.00	0.00	0.12	0.50	0.00	0.00	0.38	9.00	0.00
Z	0.38	0.50	0.12	0.75	0.25	0.25	0.62	0.88	0.38	6.50

ELS

Diagonal SD =±1.41 Non-diagonal SD =±0.61

	C	D	H	K	N	O	R	S	V	Z
C	6.50	0.75	0.12	0.25	0.25	1.00	0.50	0.50	0.12	0.00
D	1.00	6.00	0.00	0.00	0.50	1.38	0.38	0.50	0.12	0.12
H	0.12	0.12	6.50	0.00	2.38	0.00	0.62	0.00	0.12	0.12
K	0.50	0.00	0.38	6.12	0.75	0.00	2.12	0.00	0.00	0.00
N	0.38	0.12	0.62	0.62	6.12	0.12	1.50	0.25	0.25	0.12
O	1.38	0.88	0.00	0.00	0.25	6.75	0.12	0.62	0.00	0.00
R	0.00	0.12	0.38	0.00	1.62	0.25	6.75	0.62	0.00	0.00
S	0.25	0.62	0.38	0.62	0.25	0.75	0.75	6.12	0.12	0.12
V	0.12	0.00	0.00	0.50	0.88	0.25	0.25	0.50	7.00	0.12
Z	0.12	0.50	0.25	1.12	0.12	0.12	0.50	1.00	0.12	6.25

(b)

Diagonal SD =±1.69 Non-diagonal SD =±0.68

	C	D	H	K	N	O	R	S	V	Z
C	5.50	0.50	0.38	0.12	0.12	1.25	0.62	0.88	0.00	0.25
D	0.62	6.25	0.88	0.38	0.62	0.25	0.25	0.25	0.25	0.12
H	0.00	0.00	8.25	0.00	0.88	0.00	0.25	0.00	0.00	0.12
K	0.25	0.00	0.25	6.38	0.75	0.25	1.50	0.12	0.00	0.00
N	0.12	0.12	0.25	0.50	7.88	0.00	0.75	0.25	0.00	0.00
O	2.00	0.62	0.38	0.12	0.38	5.38	0.50	0.12	0.00	0.12
R	0.12	0.38	1.00	0.12	0.75	0.12	7.38	0.12	0.00	0.00
S	0.75	1.00	0.38	0.12	0.00	0.25	1.38	5.88	0.00	0.25
V	0.00	0.12	0.38	0.50	0.38	0.00	0.25	0.12	8.38	0.25
Z	0.50	0.12	0.25	0.62	0.12	0.12	0.75	0.25	0.00	7.38

Diagonal SD =±1.62 Non-diagonal SD =±0.75

	C	D	H	K	N	O	R	S	V	Z
C	5.50	0.75	0.38	0.25	0.12	1.50	0.88	0.50	0.00	0.25
D	0.75	6.62	0.62	0.25	0.25	0.62	0.25	0.00	0.25	0.12
H	0.25	0.25	5.75	0.75	1.88	0.12	0.38	0.25	0.00	0.25
K	0.38	0.00	0.38	7.00	0.50	0.12	1.25	0.00	0.00	0.12
N	0.38	0.38	1.25	0.50	6.38	0.12	0.62	0.50	0.00	0.00
O	1.62	0.75	0.00	0.12	0.50	6.62	0.62	0.12	0.00	0.12
R	0.25	0.50	1.25	0.50	0.88	0.25	6.25	0.38	0.00	0.00
S	0.88	1.12	0.25	0.12	0.00	0.12	1.00	6.25	0.00	0.12
V	0.00	0.00	0.62	0.75	0.75	0.12	1.12	0.38	5.50	0.25
Z	0.50	0.25	0.25	0.62	0.12	0.25	0.62	0.38	0.00	6.88

(c)

Diagonal SD =±1.74 Non-diagonal SD =±0.7

	C	D	H	K	N	O	R	S	V	Z
C	5.75	0.38	0.12	0.38	0.12	1.50	0.62	0.88	0.12	0.25
D	1.12	6.38	0.12	0.00	0.25	0.88	0.75	0.50	0.12	0.00
H	0.00	0.38	7.88	0.00	0.88	0.12	0.88	0.00	0.00	0.25
K	0.00	0.25	0.00	6.62	0.38	0.12	2.38	0.12	0.12	0.12
N	0.12	0.12	1.62	0.25	6.75	0.00	0.75	0.12	0.00	0.00
O	1.50	0.88	0.00	0.12	0.25	4.50	1.38	0.50	0.00	0.00
R	0.12	0.62	0.62	0.38	1.00	0.12	7.00	0.12	0.00	0.00
S	0.25	1.50	0.00	0.25	0.00	0.25	1.00	6.25	0.38	0.12
V	0.00	0.00	0.25	0.50	0.12	0.12	0.38	0.00	8.12	0.50
Z	0.25	0.12	0.12	0.00	0.12	0.25	0.38	0.12	0.12	8.12

Diagonal SD =±1.59 Non-diagonal SD =±0.69

	C	D	H	K	N	O	R	S	V	Z
C	6.25	0.62	0.12	0.12	0.12	1.75	0.25	0.50	0.12	0.12
D	1.25	6.50	0.25	0.00	0.25	0.38	0.88	0.38	0.12	0.12
H	0.38	0.50	6.38	0.12	0.88	0.12	1.00	0.00	0.12	0.50
K	0.00	0.12	0.00	6.62	0.25	0.00	2.25	0.25	0.12	0.12
N	0.25	0.25	1.88	0.62	5.25	0.00	1.12	0.12	0.00	0.12
O	1.62	0.50	0.00	0.00	0.12	6.25	1.38	0.00	0.00	0.00
R	0.12	0.88	0.75	0.25	1.12	0.12	6.50	0.25	0.12	0.12
S	0.38	1.25	0.00	0.25	0.25	0.12	0.75	5.88	0.12	0.25
V	0.12	0.00	0.50	1.25	0.12	0.00	1.12	0.25	6.12	0.75
Z	0.38	0.12	0.12	0.25	0.25	0.38	0.75	0.38	0.25	6.62

Fig. 4. shows the mean ± SD of expected confusion matrices for the two conditions (ESS and ELS) at the (a) central, (b) upper and (c) lower visual field. The greyscale illustrates the frequency of letter responses, where darker cells show higher frequencies. The diagonal cells represent correct responses, whereas the non-diagonal cells represent incorrect responses. The SD is calculated as the square root of the average of the variances of the cells for diagonal and non-diagonal values separately.

by the difference in the relative legibility of the letters, response biases and/or letter similarities. To investigate the potential effect of these three factors on the letter detection thresholds, response biases and letter similarities were computed using Luce's choice model (Luce, 1963). This model attempts to disentangle the response factor that is sensory-independent (*i.e.* response biases towards some letters) from the sensory-dependent response factor (*i.e.* similarities between certain letters). Luce's choice model was used to estimate the response biases (expressed in response bias vector) (Eq. (3)) and letter similarities (Eq. (4)). The model predications are presented as similarity matrices capturing the similarity between each pair of letters parameters. These parameters were calculated from the matrices of the maximum likelihood estimates which resulted from the model fit (see Appendix). According to the Luce's model,

$$\beta_j = \frac{1}{\sum_{k=1}^{N}\sqrt{\frac{\widehat{P}_{jk}\times\widehat{P}_{kk}}{\widehat{P}_{kj}\times\widehat{P}_{jj}}}} \tag{3}$$

and

$$\eta_{ij} = \sqrt{\frac{\widehat{P}_{ij}\times\widehat{P}_{ji}}{\widehat{P}_{ii}\times\widehat{P}_{jj}}} \tag{4}$$

where the variable β in Eq. (3) denotes the response bias parameter for the letter j. N is the number of letters (10 letters). η is the similarity parameter of each cell between the letter i and the letter j. $\widehat{P}$ is the expected value in each cell obtained from the maximum likelihood estimates matrix.

3.2. Response bias

Table 2 shows the mean (±SD) response biases; β parameter in Eq. (3). Values of 0.1 would imply guessing (*i.e.* no-bias value). Letters with values higher than 0.1 are considered to be relatively biased on the expense of those with values lower than 0.1. Fig. 5 shows the response biases (mean with 95% CI) for individual letters at the two conditions, ESS and ELS, depicted for central, upper and lower visual field locations. For letters presented at ESS, the β values for the letters C, K, O, V, Z at central, upper and lower, the letter D at central and lower, and the letter S at upper and lower visual field locations were significantly lower than the guessing rate, indicating that the response biases were towards the

Table 2
shows the mean of individual bias parameters (Mean ± SD*)* for letters for the two conditions (ESS and ELS) at central and paracentral locations.

	β for letters presented at ESS			*β* for letters presented at ELS		
	Central	Upper field (3°)	Lower field (3°)	Central	Upper field (3°)	Lower field (3°)
C	0.037 ± 0.020	0.047 ± 0.036	0.027 ± 0.032	0.118 ± 0.005	0.105 ± 0.087	0.120 ± 0.098
D	0.049 ± 0.036	0.070 ± 0.074	0.056 ± 0.036	0.099 ± 0.011	0.125 ± 0.100	0.130 ± 0.103
H	0.121 ± 0.075	0.151 ± 0.084	0.118 ± 0.078	0.084 ± 0.052	0.101 ± 0.068	0.078 ± 0.043
K	0.043 ± 0.044	0.056 ± 0.050	0.047 ± 0.038	0.090 ± 0.069	0.140 ± 0.078	0.074 ± 0.070
N	0.234 ± 0.085	0.115 ± 0.081	0.169 ± 0.142	0.150 ± 0.071	0.121 ± 0.062	0.079 ± 0.081
O	0.031 ± 0.029	0.020 ± 0.015	0.032 ± 0.032	0.104 ± 0.032	0.102 ± 0.069	0.072 ± 0.038
R	0.165 ± 0.063	0.110 ± 0.075	0.133 ± 0.079	0.146 ± 0.028	0.134 ± 0.077	0.153 ± 0.034
S	0.084 ± 0.076	0.036 ± 0.038	0.047 ± 0.028	0.090 ± 0.060	0.064 ± 0.049	0.076 ± 0.057
V	0.028 ± 0.010	0.015 ± 0.011	0.048 ± 0.042	0.038 ± 0.036	0.017 ± 0.011	0.028 ± 0.027
Z	0.026 ± 0.023	0.041 ± 0.042	0.046 ± 0.029	0.017 ± 0.018	0.011 ± 0.005	0.094 ± 0.083

remaining letters (mainly H, N, R). The pattern of the response biases in the central location was found to be significantly correlated with the pattern of response biases at the upper ($r = 0.77$, $n = 10$, $p < .01$) and lower ($r = 0.96$, $n = 10$, $p < .0001$) visual field locations. The patterns of the response biases at upper and lower visual field locations were also significantly correlated ($r = 0.85$, $n = 10$, $p < .01$). Results show that the biased letters (H, N, R) were associated with letters with low resolution thresholds (or high legibility) at central, upper and lower visual field locations. These three letters were among the five letters with low resolution thresholds. Unlike the other low resolution thresholds letters (H, N, R), the letters V and Z were associated with low bias parameters. After excluding the letters V and Z, it was found that the correlations between the response biases parameters and the individual thresholds were significantly and negatively correlated at central ($r = -0.95$, $n = 8$, $p < .001$), upper ($r = -0.94$, $n = 8$, $p < .001$) and lower ($r = -0.76$, $n = 8$, $p < .05$) visual field locations.

For the ELS condition, at least seven letters showed β values distributed around 0.1 (guessing; Fig. 5) at central, upper and lower visual field locations. The response biases were found to be mainly towards the letter R at these locations on the expense of the letter V at central, upper and lower and on the expense of the letter Z at central and upper visual field locations. The pattern of the response biases of central location was found to be significantly correlated with the pattern of response biases at the upper ($r = 0.75$, $n = 10$, $p < .05$) and at the lower ($r = 0.70$, $n = 10$, $p < .05$) visual field locations. The patterns of the response biases at upper and lower visual field locations were also significantly correlated ($r = 0.70$, $n = 10$, $p < .05$). After excluding the letters V and Z, (which showed almost similar behaviour as in ESS condition) it was found that the correlations between the β values and the individual thresholds were not significant at central ($r = -0.60$, $n = 8$, $p = .12$), upper ($r = -0.25$, $n = 8$, $p = .55$) and lower ($r = -0.31$, $n = 8$, $p = .45$) visual field locations.

These results suggest that the differences in individual thresholds (*i.e.*, relative legibility) induced response biases towards the letters with low resolution thresholds (*i.e.* high legibility, such as H, N, R) and induced biases against the letters with high resolution thresholds (*i.e.* low legibility, such as C, D, K, O) in the ESS condition.

3.3. *Similarity*

The mean (±SD) of letter similarities for letter pairs (η parameters in Eq. (4)) at the central and paracentral locations are shown in Fig. 6 as similarity matrices for the two conditions (ESS and ELS). η values of 1 represents completely identical letters (not shown in Fig. 6). For the ESS condition, the most confused pairs (*i.e.* with η parameter closest to 1) were found to be O-D at central, O-C at upper and S-D at lower visual field locations. For further analysis, the confusability of each letter with the remaining letters was calculated as the average of the η parameter (Fig. 7). The letter D at the central and the lower and C at the upper visual field locations showed the highest confusability for the ESS condition, whereas the letters V at the central and Z at the upper and the lower visual field location showed the lowest confusability (Table 3). The pattern of the confusability of individual letters of the central locations was significantly correlated with the pattern of confusability at the upper ($r = 0.89$, $n = 10$, $p < .001$) and lower ($r = 0.90$, $n = 10$, $p < .001$) visual fields locations. The pattern of confusability at the upper and lower visual fields were also correlated ($r = 0.88$, $n = 10$, $p < .001$). Despite some preference towards the confusion among curved letters (O, D, C, S), the confusability did not show significant correlations with the individual thresholds at central ($r = 0.60$, $n = 10$, $p = .066$), upper ($r = 0.55$, $n = 10$, $p = .10$) and lower ($r = 0.60$, $n = 8$, $p = .08$) visual field locations. However, for the letters presented at ELS condition, the most confused pairs were found to be R-N at central, N–H at upper and lower visual field locations. The letters N at the central and the upper and R at the lower visual field location showed the highest confusability. Additionally, the letter V at the central, the upper and the lower visual field locations showed the lowest confusability (Table 3). The pattern of the confusability of individual letters of the central location was significantly correlated with the pattern of confusability at the upper ($r = 0.92$, $n = 10$, $p < .001$) and lower ($r = 0.70$, $n = 10$, $p < .05$) visual fields locations. The pattern of confusability at the upper and lower visual fields were also significantly correlated ($r = 0.75$, $n = 10$, $p < .05$). The confusability did not show significant correlations (or association) with

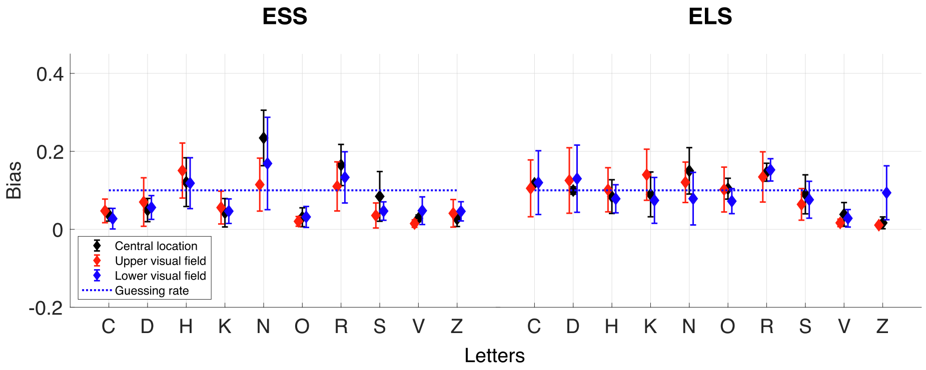
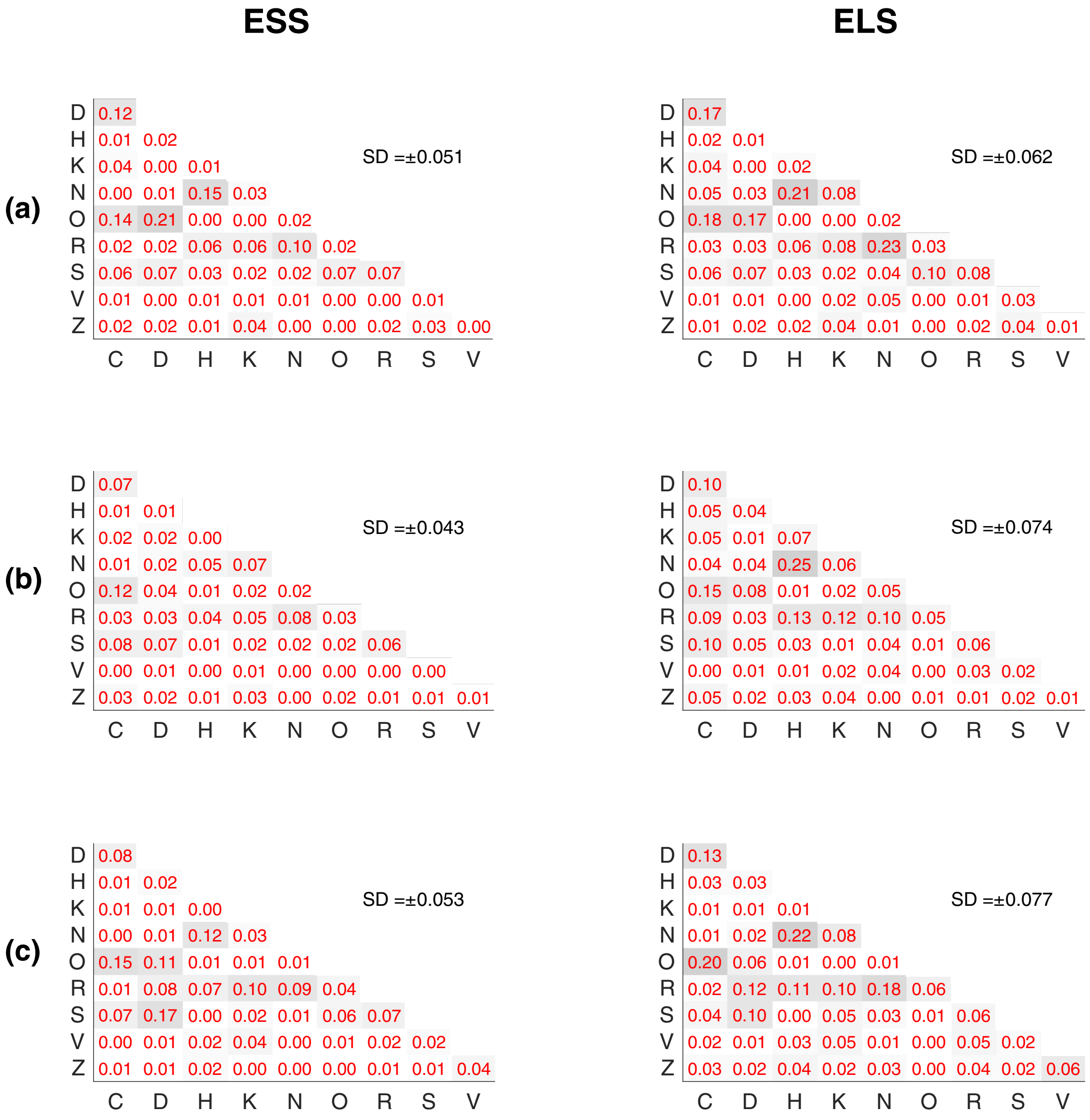


Fig. 5. shows the mean response biases for individual letters calculated by Luce choice model for the two conditions (ESS and ELS) at the central, upper (3°) and lower (3°) visual field. The horizontal dot line is the β parameters of the guessing rate or no-bias (0.1).



(a) ESS — SD =±0.051

	C	D	H	K	N	O	R	S	V
D	0.12								
H	0.01	0.02							
K	0.04	0.00	0.01						
N	0.00	0.01	0.15	0.03					
O	0.14	0.21	0.00	0.00	0.02				
R	0.02	0.02	0.06	0.06	0.10	0.02			
S	0.06	0.07	0.03	0.02	0.02	0.07	0.07		
V	0.01	0.00	0.01	0.01	0.01	0.00	0.00	0.01	
Z	0.02	0.02	0.01	0.04	0.00	0.00	0.02	0.03	0.00

(a) ELS — SD =±0.062

	C	D	H	K	N	O	R	S	V
D	0.17								
H	0.02	0.01							
K	0.04	0.00	0.02						
N	0.05	0.03	0.21	0.08					
O	0.18	0.17	0.00	0.00	0.02				
R	0.03	0.03	0.06	0.08	0.23	0.03			
S	0.06	0.07	0.03	0.02	0.04	0.10	0.08		
V	0.01	0.01	0.00	0.02	0.05	0.00	0.01	0.03	
Z	0.01	0.02	0.02	0.04	0.01	0.00	0.02	0.04	0.01

(b) ESS — SD =±0.043

	C	D	H	K	N	O	R	S	V
D	0.07								
H	0.01	0.01							
K	0.02	0.02	0.00						
N	0.01	0.02	0.05	0.07					
O	0.12	0.04	0.01	0.02	0.02				
R	0.03	0.03	0.04	0.05	0.08	0.03			
S	0.08	0.07	0.01	0.02	0.02	0.02	0.06		
V	0.00	0.01	0.00	0.01	0.00	0.00	0.00	0.00	
Z	0.03	0.02	0.01	0.03	0.00	0.02	0.01	0.01	0.01

(b) ELS — SD =±0.074

	C	D	H	K	N	O	R	S	V
D	0.10								
H	0.05	0.04							
K	0.05	0.01	0.07						
N	0.04	0.04	0.25	0.06					
O	0.15	0.08	0.01	0.02	0.05				
R	0.09	0.03	0.13	0.12	0.10	0.05			
S	0.10	0.05	0.03	0.01	0.04	0.01	0.06		
V	0.00	0.01	0.01	0.02	0.04	0.00	0.03	0.02	
Z	0.05	0.02	0.03	0.04	0.00	0.01	0.01	0.02	0.01

(c) ESS — SD =±0.053

	C	D	H	K	N	O	R	S	V
D	0.08								
H	0.01	0.02							
K	0.01	0.01	0.00						
N	0.00	0.01	0.12	0.03					
O	0.15	0.11	0.01	0.01	0.01				
R	0.01	0.08	0.07	0.10	0.09	0.04			
S	0.07	0.17	0.00	0.02	0.01	0.06	0.07		
V	0.00	0.01	0.02	0.04	0.00	0.01	0.02	0.02	
Z	0.01	0.01	0.02	0.00	0.00	0.00	0.01	0.01	0.04

(c) ELS — SD =±0.077

	C	D	H	K	N	O	R	S	V
D	0.13								
H	0.03	0.03							
K	0.01	0.01	0.01						
N	0.01	0.02	0.22	0.08					
O	0.20	0.06	0.01	0.00	0.01				
R	0.02	0.12	0.11	0.10	0.18	0.06			
S	0.04	0.10	0.00	0.05	0.03	0.01	0.06		
V	0.02	0.01	0.03	0.05	0.01	0.00	0.05	0.02	
Z	0.03	0.02	0.04	0.02	0.03	0.00	0.04	0.02	0.06

Fig. 6. shows the mean (±SD) of letter similarities (η parameters in Eq. (4)) for letter pairs for the two conditions (ESS and ELS) at the (a) central location, (b) upper (3°) and (c) lower (3°) visual field. SD is calculated as the square root of the average of η parameters' variances. The greyscale illustrates η parameters, where darker cells show higher η parameters.

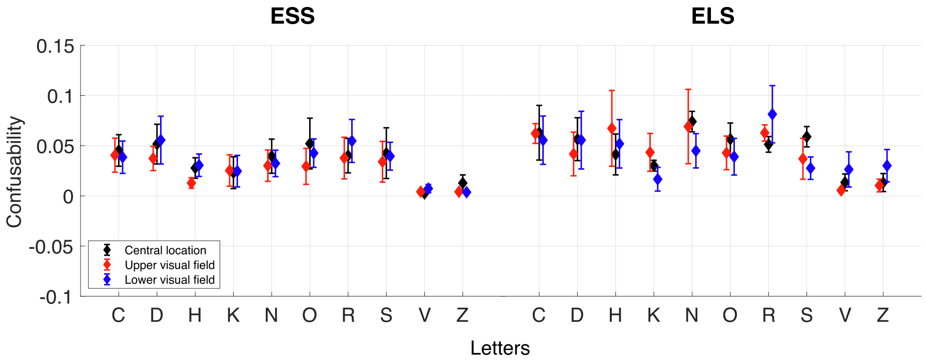


Fig. 7. shows the mean of the confusability of each individual letter for the two conditions (ESS and ELS) at the central, upper (3°) and lower (3°) visual field.

the individual thresholds at central ($r = 0.18$, $n = 10$, $p = .61$), upper ($r = 0.04$, $n = 10$, $p = .91$) and lower ($r = 0.16$, $n = 10$, $p = .65$) visual field locations.

In summary, these results suggest that differences in individual thresholds (*i.e.*, relative legibility) had a small effect on the similarity preferences. The letters with high resolution thresholds (*i.e.*, low legibility) might induce a preference towards the confusion among the curved letters (O, D, C, S). This preference has disappeared in the ELS condition.

Table 3
shows the mean of individual confusability (Mean ± SD) for the two conditions (ESS and ELS) for letters at central and paracentral locations.

	The confusability of letters presented at ESS			The confusability of letters presented at ELS		
	Central	Upper field (3°)	Lower field (3°)	Central	Upper field (3°)	Lower field (3°)
C	0.045 ± 0.019	0.041 ± 0.020	0.039 ± 0.019	0.063 ± 0.033	0.062 ± 0.012	0.055 ± 0.029
D	0.052 ± 0.024	0.037 ± 0.014	0.056 ± 0.028	0.056 ± 0.026	0.042 ± 0.026	0.056 ± 0.034
H	0.028 ± 0.012	0.013 ± 0.006	0.031 ± 0.013	0.041 ± 0.024	0.067 ± 0.045	0.052 ± 0.029
K	0.023 ± 0.019	0.025 ± 0.019	0.025 ± 0.019	0.030 ± 0.006	0.043 ± 0.023	0.017 ± 0.014
N	0.040 ± 0.020	0.030 ± 0.019	0.032 ± 0.016	0.074 ± 0.012	0.069 ± 0.044	0.045 ± 0.020
O	0.052 ± 0.030	0.029 ± 0.021	0.043 ± 0.017	0.057 ± 0.019	0.043 ± 0.020	0.039 ± 0.022
R	0.040 ± 0.021	0.038 ± 0.025	0.055 ± 0.026	0.051 ± 0.009	0.063 ± 0.010	0.081 ± 0.034
S	0.043 ± 0.030	0.034 ± 0.024	0.040 ± 0.017	0.059 ± 0.012	0.037 ± 0.024	0.028 ± 0.013
V	0.002 ± 0.002	0.004 ± 0.003	0.007 ± 0.005	0.013 ± 0.010	0.005 ± 0.002	0.026 ± 0.021
Z	0.013 ± 0.010	0.004 ± 0.002	0.004 ± 0.001	0.013 ± 0.011	0.010 ± 0.008	0.030 ± 0.019

4. Discussion

The aim of this study was to investigate the relationship between the response biases, similarity, and the individual thresholds of Sloan letters. Results showed that pooled thresholds were significantly different between the central and the paracentral visual field locations with no significant difference between the upper (3°) and lower (3°) locations. Fig. 8 shows the slope of the regression line of pooled threshold as a function of eccentricity for the current (blue) and two previous studies (green: Ludvigh, 1941; red: Hairol et al., 2015). A direct comparison of the regression lines shows a similar slope, but higher pooled thresholds measured in the current study (Fig. 8). The higher pooled thresholds in the current study could be the result of differences in study design. Hairol et al. (2015) used Sheridan Gardiner letters and unlimited viewing time, and Ludvigh (1941) used F, E, C, L, T letters and did not specify the viewing time.

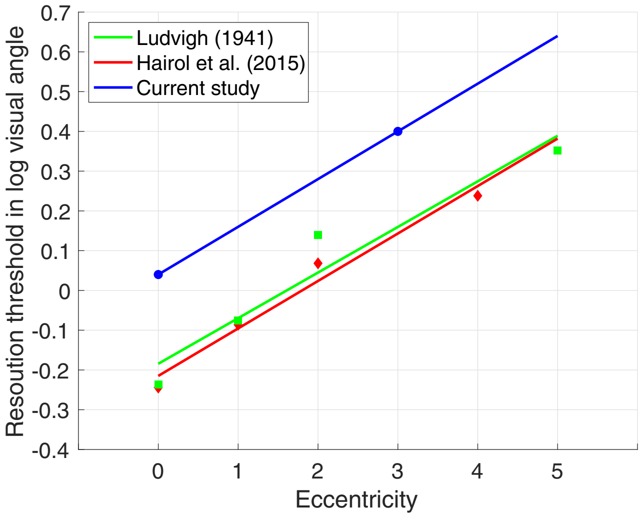


Fig. 8. shows the pooled thresholds as a function of eccentricity for two previous studies (green and red) and for the current study (blue). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Similar to previous studies, our results show significant differences (and similar pattern) of individual thresholds (Alexander et al., 1997; Reich & Bedell, 2000; Shah et al., 2012; Hamm et al., 2018). Fig. 9 shows the individual thresholds (central) compared to four previous studies (Alexander et al., 1997; Reich & Bedell, 2000; Shah et al., 2012; Hamm et al., 2018). Our results were generally similar to Alexander et al. (1997), Reich and Bedell (2000) and Shah et al. (2012) results, but different from Hamm's (2018) results, who measured lower individual thresholds for all letters. The lower individual thresholds in Hamm's (2018) study could be the consequence of using white optotypes on a black background, using interleaved QUEST staircase procedure, and a potential learning effect which could arise from the long and multiple experimental sessions (10 h per subject) (Westheimer, 2003; Hamm et al., 2018). However, despite using different psychophysical procedures[1], the pattern of the differences of individual thresholds were similar in all studies. This suggests that using different psychophysical procedures to measure the resolution thresholds has no influence on the pattern of the differences of Sloan letter individual thresholds. The current study also showed similar patterns of the differences in individual thresholds at the fovea, upper, and lower visual field locations, but with higher variability in individual thresholds in the upper and lower visual fields locations. This was consistent to what has been reported by Hairol et al. (2015).

Here we used Luce's choice model (Luce, 1963) to estimate letter similarity and response bias parameters to investigate the relationship of these parameters with the individual thresholds of Sloan letters. Our results are consistent with previous studies. Specifically, curved letters (such as C, D, O, S) were confused more frequently with each other than with the straight vertical/oblique letter (such as H, K, N, R) and vice versa. The majority of confusion pairs at the central location were found to be similar to those at paracentral locations (but with different values and ranks of η parameters). The top five similarly perceived letter pairs in term of η parameter (*e.g.* C and O is a similarly perceived letters pair) were compared to results reported previously (Table 4). Similar to

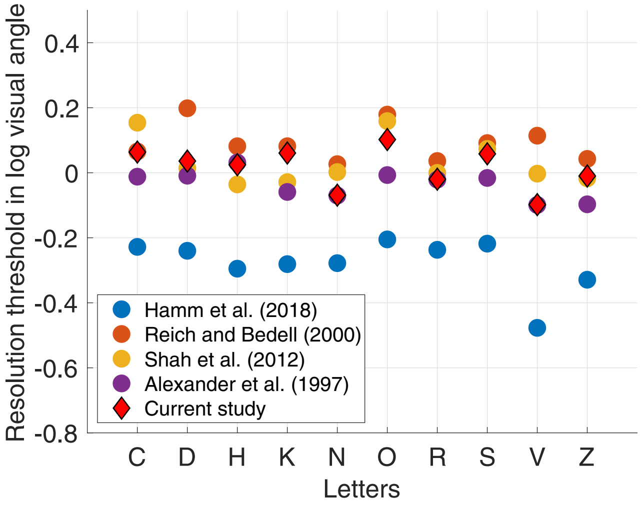


Fig. 9. shows the individual thresholds for the current and four previous studies (Alexander et al., 1997; Reich & Bedell, 2000; Shah et al., 2012; Hamm et al., 2018).

---

[1] Alexander et al. (1997), Reich and Bedell (2000) and the current study employed the method of constant stimuli, Shah et al. (2012) the method of limits, and Hamm et al. (2018) an interleaved QUEST staircase

previous studies, in the ESS condition, the preference was mostly towards confusion among curved letters (Shah et al., 2012; Reich & Bedell, 2000). Four out of five (three curved letters pairs and one straight letters pairs) in the fovea and two out of five (curved letters pairs) at the upper paracentral visual field were found to be similar to what was reported by Reich and Bedell (2000). These differences could also be due to differences in study design. In contrast to the current study, Reich and Bedell (2000) measured thresholds at 10° superior visual field location and used separate sessions for the fovea and periphery. Reich and Bedell (2000) used 25 letters which would result in different combinations of confusion pairs. Additionally, in the current study, subjects were "forced" to respond only from Sloan letters set. In this case subjects created different combinations or at least different strengths of the letter similarities (Carkeet, 2001).

In the ELS condition, the similarity parameters (and the confusability) were generally increased compared to the ESS condition. The increase was higher for letter pairs with straight vertical/oblique features such as R, N, H. In addition, the variabilities increased at three locations for similarity and confusability which implies that the confusion between letters became more random across subjects with no preference towards either of the letters' confusion sets. These results are similar to Hamm et al. (2018) who calculated the similarity from data collected near individual threshold of Sloan letters (*i.e.* ELS condition in the current study). Three out of five similarity pairs (two pairs from curved letters and one pair from straight vertical/oblique lines letters) at the fovea were found to be similar to the results of Hamm et al. (2018), but with different η parameters. As mentioned earlier, this could be the results of using reversed contrast optotypes (white optotypes on a black background) and/or employing different methods to determine the individual thresholds.

We further aimed to investigate the relationship between the letter similarities and the differences in individual thresholds. The similarity (expressed as confusability) did not show significant correlations with the individual thresholds at the central and paracentral locations. Hence, the similarity was unlikely the cause of the differences in individual thresholds. However, our results suggest that differences in individual threshold (*i.e.*, relative legibility) had a small effect in the similarity preferences. The letters with high thresholds (*i.e.* low legibility) might induce the preference towards the confusion among the curved letters (O, D, C, S) in the ESS condition. This preference was absent in the ELS condition. Nevertheless, the increase of letter similarity and variability in the ELS condition suggest that the letter similarity are a major source of "non-random" errors in the estimation of individual thresholds. These findings are consistent with the previous studies (Erdei & Fulep, 2019; Grimm et al., 1994; Mcmonnies & Ho, 1996; McMonnies & Ho, 2000; Hamm et al., 2018).

Note that for the letters V and Z, Reich and Bedell (2000) found that the letter V was confused with the letters W and Y at central and the upper visual field (10°), and the letter Z was confused with the letter T at the upper visual field (10°) with no confusion with any letter at the fovea. However, in the current study we limited the responses to Sloan letters, which might explain the very low confusability of the letters V and Z in the current study and Hamm et al. (2018).

In the ESS condition, the response biases were consistently towards the letters H, N, R across the subjects. The β parameters negatively correlated with the individual thresholds at central, upper and lower visual field locations (after excluding the letters V and Z). In the ELS condition, on the other hand, and similar to the results of Hamm et al. (2018) (Fig. 10), the response biases for at least seven letters at central, upper and lower visual field locations were at guessing rate (0.1) (except for the letter R mainly on the expense of the letters V and Z). Moreover, the correlations between the β values and the individual thresholds were no longer significant at central, upper and lower visual field locations. These findings suggest that the differences of individual thresholds substantially induced the response biases towards the letters with low resolution thresholds (*i.e.* high legibility) in the ESS condition.

In the current study, the letters V and Z showed low response biases compared to the other highly legible letters such as N, H and R. This could be due to the high legibility associated with the very low confusability of the letters V and Z. The subjects might be reluctant to call these two letters in the case of uncertainty because the letters were distinctive (highly legible with no similar letters in the Sloan letters set). This was possible especially when presenting the letters at different sizes when using the method of constant stimuli. In this case the subjects would not call these two letters unless they were very certain as these letters remained distinct at different sizes. On the other hand, for the letter R for example, which showed high legibility and confusability compared to V and Z, the subjects had a higher chance to call the letter when using the method of constant stimuli, either as correctly perceived letter or wrongly confused with other similar letters (such as N and K). This would lead subjects to overcall this letter compared to V and Z in the case of uncertainty and would consequently cause a higher response

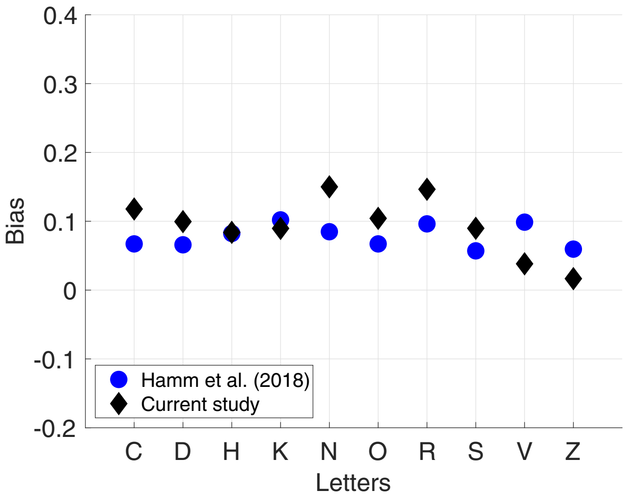


Fig. 10. shows the β parameters of individual Sloan letters at the fovea for the current study and Hamm et al. (2018).

Table 4
shows the top five similarly perceived letters pairs in the current and two previous studies (Hamm et al., 2018; Reich & Bedell, 2000).

Central location Current experiment				Hamm et al. (2018)		Reich and Bedell (2000)		Upper visual field Current experiment (3°)				Reich and Bedell (2000) (10°)	
ELS		ESS						ELS		ESS			
Letters pairs	η	Letters pairs	η	Letters pairs	η	Letters pairs	% conf.*	Letters pairs	η	Letters pairs	η	Letters pairs	% conf.*
R-N	0.23	O-D	0.21	N-H	0.14	C-O	0.17	N-H	0.25	C-O	0.12	C-O	0.24
N-H	0.21	N-H	0.15	O-D	0.12	O-D	0.16	C-O	0.15	S-C	0.08	N-H	0.20
C-O	0.18	C-O	0.14	R-K	0.11	N-H	0.12	R-H	0.13	R-N	0.08	R-D	0.16
O-D	0.17	C-D	0.12	C-O	0.09	R-K	0.10	R-K	0.12	C-D	0.07	R-K	0.13
C-D	0.17	R-N	0.10	S-C	0.05	C-D	0.09	D-C	0.10	S-D	0.07	S-C	0.13

* % conf. = The probability of confusion.

bias.

Our experiment clearly demonstrates that the response bias is substantially influenced by the relative legibility of the individual letters when letters are presented at the size of the pooled threshold (*i.e.* ESS). The response biases occur for the letters with high legibility. This might be because of the difference in the rate of errors committed for each letter. In experiments using the method of constant stimuli, each letter has the same number of presentations. In this case, high legibility letters have lower error rates than letters with low legibility when presented at the size of the pooled threshold (*i.e.* ESS). Hence, in case of uncertainty, there will be a lower chance to call the low legibility letters when presenting high legibility letters. On the other hand, when the low legibility letters are presented, in case of uncertainty there will be a higher chance to call the high legibility letters because of the higher error rates associated with the low legibility letters. This assumption is only valid when associated with some level of confusability. In the absence of confusability, the highly legible letters will be very distinctive and will not be called when uncertain, hence low response biases were observed for the letters V and Z. This assumption is also valid for the response biases calculated at individual thresholds (*i.e.* ELS). In this case, all letters have equal legibility (regardless of the size), hence equal performances lead to equal error rates for all letters. Therefore, all letters have the same chance to be called when uncertain, which causes no or minimal response biases. Note that, in current study, over-calling the highly legible letters did not seem to have a significant effect on the response biases. However, this might be the cause of the response bias observed mainly for the letter R in the ELS condition.

Our results suggest that the response biases could have significant impact on individual and pooled thresholds. Over-calling of the high legible letters increases the chance of the correct guessing compared to low legibility letters. This would result in an underestimation of the individual thresholds of high legible letters and would consequently result in lower pooled thresholds.

In conclusion, the results of the current study emphasize the importance of adopting equally legible letters to minimise the response biases and hence to avoid potential underestimation of pooled thresholds, *i.e.* an overestimation of visual acuity.

CRediT authorship contribution statement

Hatem Barhoom: Conceptualization, Methodology, Software, Writing – original draft. **Mahesh R. Joshi:** Supervision, Data curation, Writing – review & editing. **Gunnar Schmidtmann:** Visualization, Investigation, Supervision, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Luce's choice model

The fitting algorithm was originally presented by Smith (1982) and consists of two steps (Smith, 1982; Coates, 2015). The first step is to find the maximum likelihood estimate of the model. The iterative proportional fitting is used to converge the raw (confusion) matrix to the maximum likelihood estimate of the model. The starting matrix values are all ones. In order to perform the first iteration, adjustment for rows, columns and similarities are carried out successively. The adjustment for rows is performed by dividing the value of each cell by the sum of the corresponding row values of the starting matrix (ones for the first iteration) then multiplied by the marginal sum of the corresponding row values of the raw (confusion) matrix, followed by the adjustment for columns and for similarities. The resulting matrix will be the starting matrix for the second iteration. These iterations are repeated until there is no significant change in the estimated values. The resulting matrix is the maximum likelihood estimate of the model. The second step is to compute the parameters of the response bias vector and the similarity matrix from the maximum likelihood estimate of the model, according to Eqs. (3) and (4) respectively.

The MATLAB implementations to compute the response bias and similarity parameters using Luce's choice model can be downloaded from here: https://github.com/HBarhoom/Codes-

The expected confusion matrix

The expected confusion matrix at the pooled thresholds and the individual thresholds were calculated according to the following method. If a letter's threshold falls between two particular letter sizes, we used the confusion matrices of these two sizes to calculate the expected confusion matrix of the letter. First, we determined the ratio of the letter's threshold to the two enclosing letter sizes (*e.g.*, if the ratio is 0.5, we assumed that the letter's threshold is halfway between the two enclosing letter sizes). Next, we calculated the differences between the corresponding cells of the two confusion matrices of the enclosing letter sizes. Finally, we used the calculated ratio to extrapolate the expected confusion matrix at the letter's threshold level from these differences. We assumed that the central part of the psychometric function between the upper and lower asymptotes (*i.e.*, at the threshold) is approximately linear.

References

Alexander, K. R., Xie, W., & Derlacki, D. J. (1997). Visual acuity and contrast sensitivity for individual Sloan letters. *Vision Research, 37*(6), 813–819.

Anderson, R., & Thibos, L. (2004). The filtered Fourier difference spectrum predicts psychophysical letter discrimination in the peripheral retina. *Spatial Vision, 17*(1-2), 5–15.

Brainard, D. H. (1997). The psychophysics toolbox. *Spatial Vision, 10*(4), 433–436.

Carkeet, A. (2001). Modeling logMAR visual acuity scores: Effects of termination rules and alternative forced-choice options. *Optometry and Vision Science, 78*(7), 529–538.

Coates, D. R. (2015). *Quantifying crowded and uncrowded letter recognition* (Doctoral dissertation. Berkeley): University of California.

Erdei, G., & Fulep, C. (2019). U.S. Patent Application No. 16/394,388.

Grimm, W., Rassow, B., Wesemann, W., Saur, K., & Hilz, R. (1994). Correlation of optotypes with the Landolt ring – A fresh look at the comparability of optotypes. *Optometry and Vision Science, 71*(1), 6–13.

Hairol, M. I., Abd-Latif, N. A., Low, P., Lim, W. P., Aik, J. Y., & Kaur, S. (2015). Effects of foveal and eccentric viewing on the resolution and contrast thresholds of individual letters. *Psychology & Neuroscience, 8*(2), 183–192.

Hamm, L. M., Yeoman, J. P., Anstice, N., & Dakin, S. C. (2018). The Auckland Optotypes: An open-access pictogram set for measuring recognition acuity. *Jouranl of Vision, 18* (3), 13.

Kleiner, M., Brainard, D., Pelli, D., Ingling, A., Murray, R., & Broussard, C. (2007). What's new in psychtoolbox-3. *Perception, 36*(14), 1–16.

Kniestedt, C., & Stamper, R. (2003). Visual acuity and its measurement. *Ophthalmology Clinics of North America, 16*(2), 155–170.

"Detection and recognition". Handbook of mathematical psychology (Vol. 1,, (1963), 103–189.

Ludvigh, E. (1941). Extrafoveal visual acuity as measured with snellen test-letters*. *American Journal of Ophthalmology, 24*(3), 303–310.

Macmillan, N. A., & Creelman, C. D. (1990). Response bias: Characteristics of detection theory, threshold theory, and "nonparametric" indexes. *Psychological Bulletin, 107* (3), 401–413.

Mcmonnies, C. W., & Ho, A. (1996). Analysis of errors in letter acuity measurements. *Clinical and Experimental Optometry, 79*(4), 144–151.

McMonnies, C. W., & Ho, A. (2000). Letter legibility and chart equivalence. *Ophthalmic and Physiological Optics, 20*(2), 142–152.

Mueller, S. T., & Weidemann, C. T. (2012). Alphabetic letter identification: Effects of perceivability, similarity, and bias. *Acta Psychol (Amst), 139*(1), 19–37.

Pelli, D., & Robson, J. (1991). Are letters better than gratings? *Clinical Vision Sciences, 6* (5), 409–411.

Pelli, D. G. (1997). The VideoToolbox software for visual psychophysics: Transforming numbers into movies. *Spatial Vision, 10*(4), 437–442.

Prins, N., & Kingdom, F. A. A. (2018). Applying the model-comparison approach to test specific research hypotheses in psychophysical research using the palamedes toolbox. *Frontiers in Psychology, 9*(1250).

Reich, L. N., & Bedell, H. E. (2000). Relative legibility and confusions of letter acuity targets in the peripheral and central retina. *Optometry and Vision Science, 77*(5), 270–275.

Shah, N., Dakin, S. C., Redmond, T., & Anderson, R. S. (2011). Vanishing optotype letter acuity: Repeatability and effect of the number of alternatives. *Ophthalmic and Physiological Optics, 31*, 17–22.

Shah, N., Dakin, S. C., & Anderson, R. S. (2012). Effect of optical defocus on detection and recognition of vanishing optotype letters in the fovea and periphery. *Investigative ophthalmology & visual science, 53*(11), 7063–7070.

Sloan, L. L. (1959). New test Charts for the Measurement of Visual Acuity at far and Near Distances*. *American Journal of Ophthalmology, 48*(6), 807–813.

Smith, J. E. K. (1982). Recognition models evaluated: A commentary on Keren and Baggen. *Perception & Psychophysics, 31*(2), 183–189.

Strasburger, H., Rentschler, I., & Juttner, M. (2011). Peripheral vision and pattern recognition: A review. *Journal of Vision, 11*(5), 13.

Treacy, M. P., Hurst, T. P., Conway, M., Duignan, E. S., Dimitrov, B. D., Brennan, N., & Cassidy, L. (2015). The early treatment in diabetic retinopathy study chart compared with the tumbling-E and Landolt-C. *Ophthalmology, 122*(5), 1062–1063 e1061.

Westheimer, G. (2003). Visual acuity with reversed-contrast charts: I. Theoretical and psychophysical investigations. *Optometry and Vision Science, 80*(11), 745–748.